Can the informal sector affect the relationship between unemployment and output? an analysis of the Mexican case

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Abstract

A key aspect of developing countries is the existence of a large informal sector. In this paper we analyze the effect of such feature on the relationship between unemployment change and output growth for Mexico, a country characterized by the existence of a large informal sector. Following recent studies on Okun's coefficient, we first test whether the relationship between the cyclical components of unemployment and output is asymmetric. We then explore the possibility that this nonlinear relationship may be affected by changes in the informal sector. Our results indicate that there is evidence of an asymmetric relationship between the cyclical components.

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1.- Introduction

According to International Labor Organization (ILO) more than half of non agricultural employment in developing countries occurs within the informal sector. The latter has usually been associated with poor employment conditions and increasing poverty.[1] Although there is some debate about its definition and thus its magnitude, there is no question about its impact on these countries' economic performance. In the case of Mexico, for example, official statistics indicates that the latter's contribution to the overall GDP during 2003-2014 might be as high as 25 percent.

The conventional view that the labor markets in LDC were characterized by the coexistence of formal and informal markets with little or no connection between them has been displaced by a different perspective wherein the two sectors are well integrated. In the past, the predominant view was that the existence of high barriers to entry into the formal sector, in addition to the low growth, were largely responsible for the existence of a large informal sector. Within this perspective, the informal sector was seen as a macroeconomic buffer, increasing in downturns by absorbing displaced workers from formal employment and decreasing in upturns as workers re-enter formal employment (Galli and Kucera, 2003).

One of the central relationships in macroeconomics, -known as Okun's Law-, estimates the cost of unemployment in terms of output.[2] Initially, this law said that a one percentage change in unemployment would cause about three percentage points change in the opposite direction in output. Over the years, a number of studies have provided evidence against the constancy of the

[1] http://www.ilo.org/global/topics/employment-promotion/informal-economy/lang--en/index.htm. Within Latin America, for example, ILO estimates that Costa Rica has the lowest informal employment with about 30.9 % of employed workers, while Honduras and Guatemala exhibit the highest with 72.8 % and 73.6%, respectively. Mexico's informal employment is about 53.8% of non agricultural employment. (These estimates are for 2013 and for workers older than 15 years old).

[2] Or, measures the impact of output growth on unemployment change.

coefficient. It is argued that the coefficient does not remain constant because of labor markets' institutional changes, as well as technological and demographic changes that occur in most of developed countries.[3] More recently, however, some studies have evaluated whether the relationship between the cyclical components of unemployment and output is nonlinear; that is, whether the coefficient varies according to the state of the business cycle (DeLong and Summers, 1986; Courtney, 1991; Crespo, 2003; Jardin and Stephan, 2011).

For developed economies, the possibility of asymmetry is explained in terms of changes in factor substitution over the business cycle (Courtney, 1991), differences in firms' responses to the business cycle because of risk aversion (Silvapulle eta al, 2004), asymmetric responses due to hysteresis and insider/ outsider behavior (Jardin and Stephan, 2011), changes in labor force participation, differentiated sector growth rates, asymmetric adjustment costs and the role of mismatch which do not respond equally over the business cycle (Holmes and Silverstone, 2006).

The macroeconomic effect of informal sector on developing economies has been analyzed by Agénor and Azenman (1998) within a general equilibrium framework. One of the main implications of their model is that there is not a close relationship between changes in output and unemployment. In particular, a negative macroeconomic shock would induce workers employed in the formal sector to move to the informal sector with little effect on the aggregate unemployment rate. Within their model a large part of the adjustment is carried out by changes in labor productivity. Dell' Anno and Solomon (2008)[4], from a partial equilibrium perspective, also find evidence that the informal sector lowers the size of the Okun's coefficient in the case of the US economy.

[3] For a quick overview of some of the works see Silvapulle et. al. (2004) and Islas and Cortez (2013), among others.
[4] It should be noted however that Dell' Anno and Solomon (2008) do not use the term informal sector but rather "shadow economy", which, given the activities included, we take as synonymous.

Unlike previous studies on Okun's coefficient under the presence of a large informal sector, we not only argue that informal employment lowers the volatility of the relationship between output and unemployment changes but also it helps understand the transition from one regime to another over the business cycle. In this work, we propose an empirical model where the transition from one regime to the other one depends on the change in informal employment. Following recent contributions on non-linear Okun's coefficient we test three models: linear, nonlinear fixed transition probabilities (FTP) and a time varying transition probabilities (TVTP).

We find that the informal employment is an important variable in explaining the evolution of the unemployment rate in the case of an economy characterized by the existence of a large informal sector, like Mexico.

We chose Mexico for two reasons. First, it has one of the largest non agricultural informal sector in Latin America. Second, among OECD countries, it presents one of the lowest unemployment rates and yet its growth performance for the last three decades has been rather disappointing.

The paper is organized in six additional sections. Section 2, describes Mexico's labor market dynamics and in particular discusses current studies about mobility between formal and informal sectors in Mexico. Section 3, presents a brief literature review on Okun's law. We then review, in section 4, recent estimations of the Okun's coefficient for Mexico and suggests some likely mechanisms through which the informal sector may affect the former. Under the presence of a large informal sector, low growth is compatible with low unemployment. Section 5 describes the econometric models used to estimate the relationship between the cyclical components of output and unemployment. We propose three models and test whether the relationship is governed by a nonlinear relationship. Section 5 also describes the data used in the analysis. The empirical analysis is carried out in section 6, while section 7 presents some concluding remarks.

2. Mexico's labor market dynamics

Let us start with a key issue within the literature: the definition of informality. The definition of "informality" has changed over the years. Initially, informality was used to describe a situation in which poor workers were employed in small productive units because of the lack of better employment opportunities. The concept was later modified to include all workers not covered by labor legislation or social security (Tokman, 2011). Nowadays, the concept is a bit more complex for it includes a wide range of labor market activities which can be grossly clustered in two groups. On the one hand, there are the survival activities (casual jobs, temporary jobs, unpaid jobs, subsistence agriculture, multiple job holding). On the other, the rational choice activities, carried out in order to evade taxes, labor regulation and other government or institutional regulations. This includes no registration of the company.[5]

The agreement about the definition of informality, however, has been unable to create a consensus about how to measure it. Some researchers have preferred the use of workers employed in small, productive units as a proxy of informal firms, -i.e., productive units without a registry before the country's regulatory institutions-, while others have preferred the use of workers without social benefits (like health insurance and paid vacations, among others). A third group of researchers have combined both types of definitions. For example, Maloney (1998) in his study about Mexico's labor market, defines three types of informal work: i) the self-employed, .i.e., owners of informal firms with or without additional employees-; ii) the informal

[5] See World Bank Group (http://lnweb90.worldbank.org/eca/eca.nsf/1f3aa35cab9dea4f85256a77004e4ef4/2e4ede543787a0c085256a940073f4e4?OpenDocument).

salaried, which are the people working in informal firms-; iii) contract workers, are those who do not receive a regular wage or salary, but who are paid as a percentage, by piece on commission or fixed contract and are often connected to larger firms. Gong, et. al. (2000) takes workers employed in small productive units (less than 5 workers). Calderon (2000), in turn, considers informal sector as workers who are not registered in the social security institutions, whereas Bosch and Maloney (2007) include both workers in small firms and those who are uncovered by labor legislation. Galli and Kucera (2003), in addition to employment in small firms, also include self employed and domestic workers.

There are two important questions researchers have tackled recently when discussing developing and emerging economies' labor markets. First, whether their markets can be characterized as a segmented one or not. Second, the extent to which their labor markets are flexible to adjust to output shocks. Maloney (1998), for instance, provides some evidence against the segmented labor market hypothesis in the case of Mexico by estimating the transition probabilities between the formal and informal sectors. In a longitudinal analysis of three cohorts of workers[6], he finds a significant transition probability from the different types of informality to formal employment (and vice versa). He estimates that the transition probabilities from informal salaried, contract workers and self employed to formal salaried is about 42 percent; whereas the transition probabilities from the formal salaried to these different types of informality are 6%, 4% and 5%, respectively; that is, the flow from informal to formal employment is much higher than from the formal to informal.[7] Rodriguez-Oreggia (2007) estimates the transition probability

[6] The cohorts were taken from the National Urban Employment Survey 1990:q3 thru 1992:q2. The cohorts of workers were: 1990q3-1991q3, 1991q1-1992q1, and 1991q2-1992q2.
[7] The analysis includes male workers, 16-65 with high school education or less in 16 metropolitan areas. He uses two definitions of informal employment: unprotected labor or people working in micro-enterprises.

from informal to formal employment and vice versa considering different time periods. He finds that the former is higher than the transition from formal to informal employment.

Galli and Kucera (2003), on the other hand, argue than in accordance with the buffer hypothesis, informal employment is counter cyclical. However, given the different types of informal employment, one should not expect all of them to respond in the same way to business cycles. For example, informality that responds to a survival strategy is counter cyclical, while voluntary self employment is likely to be procyclical. Similarly, if firms decide to increase the amount of subcontracting or outsourcing during expansions, then informal employment is procyclical, whereas it is countercyclical if firms decide to reduce this type of employment during expansions.

Alcaraz, et al (2015) estimate the relative prevalence between voluntary and involuntary workers by using a model of self selection. They find that between 10 to 20 percent of informal workers would prefer a formal job, but they also find that entry barrier to formal employment is statistically significant. They confirm the existence of segmentation, where both formal and informal employment are somewhat integrated.

This phenomenon, however, is not exclusive of the Mexican economy. Bosch and Maloney (2007), for example, find the same pattern for Brazil and Argentina as well. As a matter of fact, they identify other similarities among their labor markets and Mexico's. First, both unemployment and informality are counter-cyclical; i. e., they both increase during recessions and decline during expansions. Second, the transitions from informality to formality, and vice versa, are pro-cyclical, largely explained by the large transition from formal employment to self-employment.

Funkhouser (1997), in a study about El Salvador's labor market estimates the transition along gender characteristics. He found that there was a fair amount of mobility between formal and informal sector. Male workers, initially in informal employment, 7.3 % moved into formal employment, whereas the transition from formal to informal employment was about 4.7%. In the case of female workers, the percentages were lower: 2.1 % and 3.3 % respectively.

Not long ago, Mexico's labor market was classified as a very rigid one. Heckman and Pagés (2000) and Gil et al (2001), among others, argued that Mexico's labor market was heavily regulated by laws that impeded employment creation. During recession and because of the rigidity of the federal labor law, it was costly and difficult for firms to lay-off workers, while during expansions, firms would hesitate to hire new workers because of the high costs involved in their hiring; in particular because of the provision of social benefits (health insurance, housing loans, paid vacations, and so on). This type of market rigidity explained why output changes could result in low unemployment changes, as Gonzales-Anaya, (2002) argued.

Recent studies however have noted the significant changes that Mexico's labor market has gone thru. In particular, the significant growth of informal employment. The existence of a large informal market with significant labor mobility between formal and informal employment suggests that Mexico's labor market can be characterized as a hybrid market, wherein firms in the formal sector are increasingly using different employment schemes looking for reductions in their labor costs (Contreras, 2000; De la Garza, 20005).

Furthermore, given the size of the informal employment and the barriers to entry to formal employment, one would expect that, for instance, foreseen output decline would induce firms to reduce their employment prospects[8]. This, in turn, would induce workers to either take an

[8] In some cases, firms would lower the number of full time workers and increase the number of part time jobs under the subcontracting scheme.

informal job or become unemployed. By the same token, output expansions would induce firms to increase their hiring using the flexible schemes available to them. Informal employment would decline and so would unemployment. It should be pointed out that in either case, unemployment changes would be lower than in the case when informal sector would not exist. This can be seen by the significant transition probability from formal to informal employment instead of a much higher transition from formal employment to unemployment.

3. A few notes on Okun's coefficient

Since Okun's innovative paper, the literature about the relationship between output growth and unemployment change has grown extensively. The existing literature provides support for the empirical validity of the tradeoff between these two variables, although there is vast evidence that the magnitude of the Okun's coefficient varies significantly within a country (over time) and across countries (see, for instance, Silvapulle, et. al., 2004; Lee, 2000; and Harris and Silverstone, 2001, among others). These studies represent a radical departure from early studies which assumed that the coefficient was stable and reliable over time (Gordon, 1984).

The non constancy of the coefficient has been attributed to a number of factors. From an accounting perspective, several authors have noted that the size of the coefficient depends on the evolution of variables such as technology, working hours, utilization rate of capital (Prachowny, 1993), factor substitution (Courtney, 1991). Other studies have noted that the coefficient is also sensitive to model specification, which includes the form of the model, -static versus dynamic-, and to the detrending method to remove non stationarity: the first difference model versus the gap model (Crespo, 2003).[9]

[9] In the first difference model, output and unemployment are expressed in first difference (growth rates), while in the gap model they are taken as the deviations from their long term trend. Within the latter case, a new question

One of the earliest studies about a nonlinear relationship between output growth and unemployment changes is Courtney (1991). Following a long tradition of research on US business cycles, Courtney finds that the Okun's coefficient is dependent on the state of the cycle. In particular, he finds that the effect of output growth on unemployment change is stronger in contractions than in expansions. He further argues that unobserved labor hoarding may be an important determinant of state dependence in Okun's law. In particular, he sustains that it is the substitution between employee's hours and his/her effort what explained the asymmetric behavior of employment over the business cycle.

A priori we cannot say in which of the two regimes, -expansions or recessions-, the impact of output change would be stronger on unemployment change. There are two contending explanations. On the one hand, there is the view which states that when the economy is contracting, firms respond very quickly by laying off workers as the economy goes into a recession. As the recession ends, firms fear the possibility that recovery may not last long, which induce them to adjust productivity and/or the number of hours rather than the number of workers (Jardin and Stephan, 2011). This asymmetric behavior thus explains why the effect of output growth on unemployment is stronger during recessions than during expansions. The contrasting view argues that firms are unwilling to lay off workers during recessions because of the high costs involved (due to labor laws) and because their investment in workers' trainings would be lost. It is further argued that during recoveries these firms would hire more workers because there are low institutional restrictions. From this perspective, one would expect the impact of output growth on unemployment during expansions be stronger than during recessions.

emerges: which filter to use. Lee (2000), for example, in a comparative analysis across developed economies evaluates the stability and robustness of the law given that many European countries had faced changes in their labor market institutions.

There were several additional conclusions drawn from Jardin and Stephan's study. First, even though the results were qualitatively similar, he found significant quantitatively differences across countries. Second, there were stronger evidences that the coefficient did not remain constant over time across countries when using the gap model as opposed to the first difference model. Third, he found structural change somewhere in the early 1970s in the majority of European countries with the exception of Austria and Canada. Fourth, the coefficient is much lower in most of European countries than that of the US economy.

Harris and Silverstone (2001) further extend the analysis of asymmetry for key OECD countries by estimating the coefficient in the long and in the short run.[10] They find that the long run coefficient lies between -0.39 and -0.5, with the UK and Japan as outliers. In the short run, they find evidence that unemployment adjusts asymmetrically to output growth; specifically, it adjusts in the expected manner during the downturn of the business cycle. They did not find reliable evidence about the response of output to changes in unemployment. In terms of policy implications, a nonlinear Okun's law would suggest an asymmetric policy response is required to reduce output fluctuations. In some cases, unemployment changes would do the job, whereas in other cases, price adjustments are required to achieve equilibrium.

Crespo (2003) and Silvapulle et al (2004), among others, found that regardless of the filtering technique, nonlinear models explain better the relationship between the cyclical components of output and unemployment than linear models. They both find that the impact of output on unemployment is stronger during recessions than during expansions.[11]

[10] The sample included Australia, Japan, New Zealand, UK, USA, Canada and Germany. They used the Engel-Granger methodology for co-integrated series, wherein the error correction term is adjusted to incorporate asymmetry. Their procedure involved estimating the threshold points that minimized the sum squared residuals through a grid search.

[11] Crespo (2003), for example, applies both the Hodrick-Prescott (HP) filter to each series individually and the bivariate structural time series model proposed by Harvey (1989). Silvapulle et al (2004), in turn, also use Harvey's bi-variate methodology to detrend the series.

The works reviewed so far estimate the asymmetric Okun's coefficient following a deterministic approach in the sense that it is assumed that the change between regimes is an exogenous and deterministic event. Since the early 2000s, however, the latter view was surpassed by a new perspective wherein not only the switch from one state to another is measured by the transition probability between the two states, but also the change in regime is triggered by the level of the output gap. Moreover, the level of the output gap that induces the change in regime is estimated within the model.

4. Informality and Okun's coefficient: the case of Mexico

Neither there are many papers that estimate the Okun's coefficient for Mexico nor there are consensus about the actual size of the coefficient. Chavarin, (2001), for example, estimated that the coefficient was close to Okun's original calculations for the US economy. [12] Gonzales-Anaya, (2002) and Islas and Cortez, (2013)[13], however, find much smaller coefficients. The smaller coefficients represent a puzzle for one would have expected that given the increasing flexibility of Mexico's labor market since the mid-nineties, the effect of output growth on unemployment would have been much higher.

It is not clear whether higher labor market flexibility would necessarily be reflected by higher employment fluctuations however. There are two answers to the question about the likely effects of higher labor market flexibility of output fluctuations on unemployment fluctuations. If flexibility involves employment flexibility, i. e., the possibility of firms to hire and fire workers

[12] Chavarin (2001) estimated that a one percentage change in unemployment would cost around 2.7% in output change. On the other hand, a one percentage change in output was associated with a 0.3% variation in unemployment in the opposite direction.

[13] Islas and Cortez (2013) estimate that a 1 percentage point change in output is associated with a -0.5 percentage change in unemployment, while a 1 percentage point change in unemployment is associated with a -1.66 percentage change in output.

according to their production needs, then, the answer is yes: we should be able to see a higher effect of output fluctuations on unemployment fluctuations. Yet, if flexibility involves the possibility to contract workers under flexible employment schemes, then the impact of output fluctuations on unemployment fluctuations would be rather small. In this case, output fluctuations would induce higher workers' mobility across the different types of jobs. Unemployment rate would still be affected by output fluctuations but it will lower[14].

This is where informal sector comes in. It has been recognized that several of these types of jobs are actually informal jobs in the sense that they do not provide the basic social benefits that a formal employment does. If in addition we take into account employment in informal firms, then, cyclical output would have an even lower effect on cyclical unemployment. The evidence presented in a previous section clearly emphasized that there is a significant transition from formal to informal employment. Hence, in the presence of a large informal sector, the effect of cyclical output on cyclical employment would be rather small. A result already noted by Agénor and Azenna (1998) and Dell' Anno and Solomon (2008).

From a partial equilibrium perspective, we propose some basic ideas that would allow us estimate the impact of the informal sector on the transition from one regime to another. Our starting point is the identity that says that Economically Active Population (EAP) is the sum of employed and unemployed workers. Expressed as a proportion of EAP, we obtain

$$e + u = 1 \tag{1}$$

We assume that there are two types of firms: formal and informal. Informal firms are those who do not pay taxes or are not legally registered before authorities. Thus, total employment is the sum of people working in formal and informal firms.

[14] Gong, et al (2000) found evidence that during recessions the transition probability from both formal and informal employment to unemployment increases with respect to periods of expansion.

$$e = f + i \qquad (2)$$

We further assume that both formal and informal employment depend on output. An increase in output induces formal firms to increase their labor demand, while it induces a decline in informal employment. Informal employment is thus a residual, i.e., it depends on the number of workers no taken by formal firms. Hence,

$$f = f(y) \qquad (3)$$

$$i = i(y, f(y)) \qquad (4)$$

Where

$f_y > 0$; $i_y < 0$; are the sensitivity of formal and informal employment to output, respectively. The latter is in accordance with the assumption that informal employment is counter cyclical; in other words, we assume that a large proportion of informal employment responds to the survival strategy rather than to the voluntary one.[15]

$i_f f_y < 0$ then formal and informal output are substitutes

Using equations (1) thru (4) we obtain

$$u = 1 - f(y) - i(y, f) \qquad (5)$$

Taking total derivative of (5) we obtain

$$du = -f_y dy - i_y dy - i_f f_y dy \qquad (6)$$

$$du = -[(f_y + i_y + i_f f_y)]dy \qquad (7)$$

Equation (7) indicates that the impact of output changes on unemployment changes can be decomposed into three effects: i) the direct impact on formal employment, (ii) the direct impact

[15] As argued, informality takes now different forms: subcontracting, part time jobs, by piece, and so on. For example, during the 1994-5 Mexican crisis, the number of workers working between 20-60 hrs a week declined significantly, while the number of workers working less than 20 hrs increased dramatically.

on informal employment; and iii) the indirect impact through their cross change. If the informal sector did not exist, Okun's coefficient would be measured entirely by the first term of the equation. The existence of the informal sector induces a lower coefficient since the impact of output on both formal and informal employment move in opposite directions. There is an additional indirect effect which may result in an even lower coefficient.

The evidence presented in section 2 suggests that not only the transition probabilities between formal and informal sectors are not similar, but also they are different over the business cycle. In addition, Oliveira, (2002) found that Mexico's business cycle is asymmetrical in the sense that the recessions are deeper and shorter than the expansions. It is evident then that this asymmetric behavior of output growth would have some impact on its relationship with the labor market.

It is further argued then that the relationship between du and dy depends on the economy's regime; that is, it depends on whether the economy is in the expansionary or in the recessionary state. We would expect that the size of the firms' reaction to changes in output would be different in the two regimes. In particular, we argue that the relationship between output and unemployment be stronger during recession than during expansions,

$$\left|\left(f_y + i_y + i_f f_y\right)^r\right| > \left|\left(f_y + i_y + i_f f_y\right)^e\right| \tag{8}$$

Now, in order to assess the impact of the informal employment on the transition probability of moving from one regime to the other, let us define the regime at which the economy is in terms of unemployment rate. We say the economy is in the expansionary state whenever the current unemployment is below its long run trend, while the economy is in the recessionary state when unemployment is above it.

In the following section we provide the details of the transition probability of moving from one regime to the next, as well as the estimation technique.

5. Methodology

5.1 Empirical Models

We estimate three models to measure the tradeoff between cyclical output and cyclical unemployment. The first model, Model I, assumes a lineal relationship, whereas model II considers a Markov switching regime model with fixed transition probability (FTP) and model III relaxes the assumption of a fixed transition probability by allowing a time varying transition probability model. As explained by Filardo (1994) and Diebold (1999), the Markov switching model with time varying transition probability (TVTP) is more flexible than the FTP. It recognizes systematic changes in the transition probabilities before and after turning points, it captures more complex temporal persistence and allows expected duration to vary across time. In this context, economic fundamentals and policy shocks can influence the regime transition probabilities.

First, we consider the traditional linear regression-based model proposed by Mossa (1997) and label it “Model I”. It is as follows

$$u_t^c = \alpha_0 + \beta y_t^c + \sum_{i=1}^{p} \alpha_j \, u_{t-j}^c + \varepsilon_t, \;\; \varepsilon_t \sim NID(0, \sigma^2) \qquad (9)$$

where u_t^c denotes cyclical unemployment, y_t^c denotes cyclical output. The lagged cyclical unemployment is required to remove serial correlation. Okun´s coefficient is measured by the estimated value of β, the impact coefficient, such that $\beta < 0$.

In addition to the linear regression-based model, we consider the Markov Switching FTP model to characterize the regime-dependent specification of Okun´s law, which allows for an asymmetric effect of cyclical output on cyclical unemployment. The general idea behind this class of regime-switching models is that the regression parameters depend upon a stochastic, unobservable regime variable $s_t \in \{1,2\}$. The stochastic process for generating the unobservable regime is an ergodic Markov chain defined by the transition probability $p_{ij} = \Pr\,(s_{t+1} = j|s_t = i)$ where *i,j=1 or 2*. The transition probabilities p_{ij} gives the probability that state *i* will be followed by state *j*. The transition matrix is

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} \qquad (10)$$

where p_{11} is the probability of remaining in the expansionary regime, defined as the outcome when unemployment rate is below its trend; while p_{22} is the probability of remaining in the recessionary regime, defined as the situation when unemployment rate is above its trend.

The time regime-dependent specification of Okun´s law which allow for an asymmetric effect, "Model II", is as follow:

$$u_t^c = \alpha_{0s_t} + \beta_{s_t} y_t^c + \sum_{i=1}^{p} \alpha_{j-1s_t} u_{t-j}^c + \varepsilon_t, \;\; \varepsilon_t \sim NID(0, \sigma_{s_t}^2) \qquad (11)$$

Where $\alpha_{js_t} = \alpha_{j1}, \beta_{s_t} = \beta_1, \sigma_{s_t}^2 = \sigma_1^2$ if $s_t = 1$, for *j=0,1,…,p* while $\alpha_{js_t} = \alpha_{j2}, \beta_{s_t} = \beta_2, \sigma_{s_t}^2 = \sigma_2^2$ if $s_t = 2$, for *j=0,1,…,p*.

Model II features two coefficients, namely β_1 and β_2. A priory, we expect $\beta_1, \beta_2 < 0$ where cyclical unemployment responds negatively to cyclical output in either expansionary or recessionary regimes.

Finally, we consider "Model III" which allows for the possibility of time-varying transition probabilities. For information variables in z_t, we choose the informal employment rate since we assume that it has been the main cause of variation in unemployment rate (see section 4). This is a departure from other studies which have used the output gap, or capacity utilization as the leading variable behind the time varying transition probability. Therefore, Model III considers the following time-varying transition probabilities,

$$P[s_t = 1|s_{t-1} = 1, \underline{z}_{t-1}; \underline{\delta_1}] = p(\underline{z}_{t-1}) = \Phi(\underline{z'}_{t-1}\underline{\delta_1}) \tag{12}$$

$$P[s_t = 2|s_{t-1} = 2, \underline{z}_{t-1}; \underline{\delta_2}] = q(\underline{z}_{t-1}) = \Phi(\underline{z'}_{t-1}\underline{\delta_2})$$

where $\Phi(.)$ refers to the cumulative density function of the standard normal distribution evolving as a functions of $\underline{z'}_{t-1}\underline{\delta_i}, i = 1,2$, where the *(mx1)* conditioning vector $\underline{z'}_{t-1} = \left(1, \Delta e_{inf_{t-1}}, \Delta e_{inf_{t-2}}, \ldots, \Delta e_{inf_{t-m-1}}\right)$, $\underline{\delta_i}' = (\delta_{i0}, \delta_{i1}, \ldots, \delta_{i,m-1}), i = 1,2$, and Δe_{inf_t} denotes the first difference of the informal employment. The two-point stochastic process on s_t can be summarized by the transition matrix

$$P[s_t = 1|s_{t-1} = 1, \underline{z}_{t-1}; \underline{\delta_i}] = \begin{bmatrix} p(\underline{z}_{t-1}) & 1 - p(\underline{z}_{t-1}) \\ 1 - q(\underline{z}_{t-1}) & q(\underline{z}_{t-1}) \end{bmatrix} \tag{13}$$

Where the history of the state of the informal unemployment is in $\underline{z}_{t-1}$.

In the time varying transition probability Markov switching model, transition probabilities are allowed to vary with the state of the informal employment (upswing and downswing).

Probabilities in Model III reflect the duration of the Okun relationship expressed as a regime-dependent specification. For the model specification test, we follow Engel and Hamilton (1990) proving the following hypothesis

$$H_0^{ss}: \alpha_{j1} = \alpha_{j2}, \beta_1 = \beta_2, \sigma_1^2 \neq \sigma_2^2; j = 0,1,2, \dots, p \tag{14}$$

If we cannot reject H_0^{ss}, then it is implied that the true data generating process comes from a single state, as opposed to two states. The test statistics for H_0^{ss} hypothesis is the Wald statistic, and it has the $\chi^2_{(\nu)}$. It should be noted that if we let $\sigma_1^2 = \sigma_2^2$, the test of H_0^{ss} has the so called "nuisance parameter problem". That is the parameter p_{11} *and* p_{22} are unidentified.

To estimate Models I to III, we need time series on the unobserved components u_t^c and y_t^c. The extraction of the cyclical components can be done using different methodologies, *e. g.*, by considering a single or a multiple time series setting. Here, we concentrate on a bivariate time series approach that assumes the observed time series vector is composed by an unobserved vector of trends plus a vector of cycles, and takes into account the correlation between output and unemployment cycles. Laxton and Tetlow (1992) provided a historical overview of estimation procedures of potential output and found that basically two approaches had been employed since the 1980s: (1) structural approaches that rely on a structural economic model, as in Ford and Rose (1989) and Adams and Coe (1990); (2) stochastic approaches as that underlying the Hodrick and Prescott (1980, 1997) (HP) filter. Laxton and Tetlow (1992) combined those approaches and proposed a semi-structural technique which is called the Hodrick-Prescott Multivariate filter (HPMV); *e. g.* Boone (2000) and Chagny and Lemoine (2002). However, the HPMV is not a true multivariate filter, but a multiple time series filter. This difference is akin to that of multiple regression where there is only one dependent variable to be explained by several independent ones, while multivariate regression considers several dependent variables to be explained simultaneously by one or more independent variables.

We use a different semi-structural technique that is based on a true multiple time series filtering method proposed by Guerrero et al (20016), called the Bivariate Hodrick-Prescott filter (BHP). This new method allows us to extract both trends that might share similar dynamic behaviors. One of the advantages of this method is that it employs only the first two sample moments of the variables involved. Another significant advantage is that it provides a way of deciding the value of the smoothing parameter that produces a desired percentage of smoothness for the trends. A third advantage is that it takes into account simultaneity in the estimation which corrects for likely biases. The method is described in the Appendix.

5.2 The Data

The key variables are output, unemployment and informal employment rates. Mexico's gross domestic product was collected from INEGI[16], and is on a quarterly basis in real pesos (base year = 2008). The unemployment series is the alternative unemployment rate estimated following the Bureau of Labor Statistics Methodology (BLS).[17] Labor series, -unemployment and informal employment-, come two sources: the National Survey of Urban Employment (ENEU) and the National Survey of Occupation and Employment (ENOE).

[16] INEGI is Mexico's National Institute of Statistics and Geography.

[17] See Fleck and Sorrentino (1894) and Martin (2000) for a discussion of the main differences between Mexico's INEGI methodology to estimate unemployment and that of the US Bureau of Labor Statistics. INEGI's methodology grossly underestimate unemployment rate. Although Mexico's INEGI made some adjustments to its methodology, there still remain some elements which make the Mexico's unemployment rates lower than that of the US economy.

Both the unemployment rate and the informal sector rate are estimated for the 11 largest cities in Mexico.[18] We selected those cities because were the only ones who were in both surveys (ENEU and ENOE) so that we could obtain a time series by using them. We considered workers between 16 and 75 years old. All data are quarterly, seasonally adjusted and covers the period between 1993:QI through 2015:QII. Employment in the informal sector is defined as workers who are employed in firms that do not have name or registry.[19]

Figure 1 describes the behavior of the jointly estimated cyclical components of output and unemployment that were extracted using the methodology described in the Appendix. This figure shows that the behavior of the cyclical components is consistent with the economic theory of Okun´s law, implying that the cyclical unemployment is negative only if cyclical output is positive and vice versa.

(Figure 1 around here)

6.- Empirical results

After obtaining the cyclical components u_t^c and y_t^c, we proceed to estimate both linear and nonlinear models. Table 1 presents the results for the three models.

The lag length of the autoregressive component of cyclical unemployment, p, was chosen to be the one minimizing Akaike´s Information Criteria. Having started with a maximum of six lags, the inclusion of one lagged value of u^c for model I, five for Models II and six for model III (together with one lag for Δe_{inf_t} in the TVTP model) were found to be accepted.

[18] The cities considered in the analysis are Mexico City, Guadalajara, Monterrey, Puebla, León, San Luis Potosí, Mérida, Chihuahua, Tampico, Veracruz, and Tijuana.

[19] It should be noted that this definition of informality is different from the definition of informal employment. The latter case refers to employment that is not covered by social benefits, even though the firm may be a formal business.

(Table 1 around here)

Given that we consider three different models in this work. First, we use the likelihood ratio test (LR) for the model selection, the results are summarized in Table 1. The LR statistic suggests that Modell II (The FTP regime-dependent specification of Okun´s law) is preferable to Model I (The linear specification of Okun´s law). Therefore, evidence of nonlinearity in the Okun´s law for Mexico is found. The results also indicate that the mean cyclical unemployment rate is lower in the expansionary than in the recessionary regime *(-0.0374< 0.2259).* As we mentioned before the expansionary and recessionary regimes are described as outcomes where unemployment is below and above trend, respectively. Across the two regimes, the two state-dependent Okun coefficients (β_1, β_2) are negative and significant at 1% level. Further test results in the rejection of the null hypothesis $H_0: \beta_1 = \beta_2$; such result supports the existence of asymmetric Okun´s coefficient. Cyclical unemployment is more responsive to contemporaneous cyclical output when the former is in the recessionary regime. Results indicate that a given decrease of 1% in cyclical output is accompanied by an increase of approximately 0.31% in unemployment if the system is in a recessionary regime, while an increase of 1% in cyclical output when the system is in an expansionary regime reduces unemployment by approximately 0.12%.

Furthermore, the probability p_{11} of staying in the expansionary regime at time *(t),* given that unemployment rate was in the same regime at time *(t-1),* is 0.90. The probability p_{22} of being in the recessionary regime at time *(t),* given that the unemployment rate was in the same regime at time *(t-1)* is 0.53, smaller than p_{11}. These probability values indicate that if the unemployment rate is in expansionary regime it is more likely to remain in such a regime than switch to a recessionary one. In addition, Table 1 shows that the probability of switching from an expansionary to a recessionary regime is almost 0.095, while the probability of changing from a recessionary to expansionary regime is close to 0.47, which indicates changes from recessionary to expansionary more likely than changes from expansionary to recessionary. The expected duration of regime j is defined as $1/(1 - p_{jj})$. According to this result, we found that the average length of expansionary regime is two and a half years, whereas the expected duration of a recessionary regime is approximately a half year.

We now estimate the model under transitional endogenous probabilities. Unlike other papers that use output changes, we allow the informal employment rate to explain the evolution of such probabilities. As we explained in section 4, we consider that the informal employment is one of the main causes of variation on unemployment rate.

The likelihood ratio test that compares the model of time-varying transition probabilities with the model of fixed probabilities rejects the null hypothesis of constant probabilities in favor of the TVTP model. Based on these tests, we conclude that the model with endogenous transition probabilities is the best model to explain the relationship between the cyclical components of unemployment and output.

The TVTP estimations, also shown in Table 1, validate the existence of two different states of the unemployment rate: an expansionary regime with a negative mean cyclical unemployment (-0.0567) and a recessionary regime with a positive mean cyclical unemployment (0.2307). In this case, the average mean cyclical unemployment in expansionary and recessionary regimes are similar to those found for the FTP case.

We observe that the sign of the explanatory variable of the transition probabilities is in accordance to economic intuition. In fact, the probability of remaining in an expansionary regime, with below-trend unemployment, increases with an increment on the informal employment rate. On the other hand, if the unemployment rate is in a recessionary regime, with above-trend unemployment, an increment on the informal employment rate decreases the probability of remaining in this regime.

As in the FTP model, across the two regimes, the two state-dependent Okun coefficients (β_1, β_2) are negative and significant at 1% level. Further test results in the rejection of the null hypothesis $H_0: \beta_1 = \beta_2$; such result supports the existence of asymmetric Okun´s coefficient. Cyclical unemployment is more responsive to contemporaneous economic growth when the former is in the recessionary regime. Results indicates that a given decrease of 1% in cyclical output is accompanied by an increase of approximately 0.26% in unemployment if unemployment rate is in the recessionary regime, while an increase of 1% in cyclical output decreases unemployment by approximately 0.10%, if unemployment is in the expansionary regime. Our results indicate that by allowing the informal employment to explain the evolution of transition probabilities, Okun´s coefficient estimates are smaller than the estimated by the FTP model. This corroborates our claim that in the presence of a large informal sector, the effect of cyclical output on cyclical employment would be rather small.

Figure 2 shows the smoothed TVTP of being in a recessionary regime with above trend unemployment rate at each date in the sample obtained from the model of endogenous probabilities. The time in which the unemployment rate switched from each regime is based on $P\left(s_t = res | u_1^c, \dots, u_T^c; \widehat{\Theta}\right) \geq 0.5$. The switching between regimes is most of the time sudden, deep and sporadic. The unemployment rate stays in an expansionary regime most of the time. As we see, Figure 2 indicates four changes from expansionary to recessionary regime during the sample period. The fist occurred between 1994:QIV-1995:QIII, the period of the Mexican financial crises of 1994. The second, during 2003:QII-2005:QI, this upward trend in the Mexican unemployment rate during this period could be related to the slowdown in the Mexican economy during the same period where the economy stagnated in the quarters 2003:QII, 2003:QIII, 2004:QII, 2004:QIII and 2005:QI. The third occurred in 2008:QII when the global financial crisis started. The last switching from an expansionary to a recessionary regime occurred in 2013:Q1 and could be related to Mexico's economy performance. During 2013 the economy grew at 1.1 percentage rate, down sharply from a 3.9 percent expansion in 2012, making its weakest performance since 2009 when Mexico slumped into a deep recession.

(Figure 2 around here)

7. **Conclusions**

In this paper, we evaluate the effect of having a large informal sector on the impact of output fluctuations on unemployment fluctuations. We also consider the possibility that such effect may change over the course of Mexico's business cycle. A nonlinear specification of the relationship between cyclical unemployment and cyclical output was estimated for the Mexican economy. By using a Markov switching model with both, fixed and time varying transition probabilities, we have identified the presence of asymmetry across regimes. In particular, in the model of endogenous probabilities, we allowed the probabilities to be affected by the rate of informal employment which we consider is the main cause of variation in unemployment rate.

Our findings can be summarized as follows. First, we corroborate previous estimates that Okun's coefficient is rather low. We argue that this low coefficient is largely explained buy the existence of a large informal sector and by the high mobility between formal and informal sectors. Second, we find evidence of a nonlinear Okun's coefficient in Mexico. In particular, our results support the existence of a regime dependent Okun's parameters with a significant higher absolute value for recessions than for expansions. This implies that the cyclical unemployment is more responsive to changes in cyclical output when the former in the recessionary regime.. Third, the hypothesis of fixed probabilities can be rejected in favor of time varying transition probabilities, which mean that a better model is the one with endogenous transition probabilities. Fourth, we propose that informal employment affects the evolution of unemployment rate significantly. We find that the probability of remaining in an expansionary regime with below-trend unemployment increases with an increment on the informal employment rate, while if the unemployment rate is in a recessionary regime, with above-trend unemployment, an increment on the informal employment rate decreases the probability of remaining in this regime.

Appendix

A.1 A model for trend and cyclical components.

In this appendix, we follow Guerrero et al (2016) Bivariate HP filter (GIR from here on). Define a bivariate column vector $\mathbf{Z}_t = (y_t, u_t)'$ for $t = 1, ..., N$, where prime denotes transpose throughout. The bivariate time series under study is assumed to be given by the signal-plus-noise model

$$\mathbf{Z}_t = \boldsymbol{\tau}_t + \boldsymbol{\eta}_t \quad \text{for} \quad t = 1, ..., N, \tag{A1}$$

where $\mathbf{Z}_t$ is the observed vector, while $\boldsymbol{\tau}_t$ is also a bivariate vector denoting its trend and $\boldsymbol{\eta}_t$ is the bivariate vector denoting its cycle. Of course, as it happens with the univariate case, such a representation does not correspond with the data generation process, but it basically serves to capture the stylized facts and allows us to use very simple tools to estimate the trend with a desired percentage of smoothness, as indicated below. We are concerned here with the estimation of the trend component given the sample of observations $\{\mathbf{Z}_t\}$ for $t = 1,\ldots, N$. To do that we consider the problem of minimizing the following quadratic function

$$M(\lambda) = \sum_{t=1}^{N}(\mathbf{Z}_t - \boldsymbol{\tau}_t)' W(\mathbf{Z}_t - \boldsymbol{\tau}_t) + \lambda\sum_{t=3}^{N}\sum_{i=1}^{2}(\tau_{i,t} - 2\tau_{i,t-1} + \tau_{i,t-2})^2 \tag{A2}$$

with $\lambda > 0$ a smoothing constant that penalizes the lack of trend smoothness and W a known symmetric and positive definite matrix of constant weights. Thus, for all t and *i = 1, 2*, if we let $\lambda \to 0$ both trends approach the observed data, *i. e.*, $\tau_{i,t} \to Z_{i,t}$, and when $\lambda \to \infty$, $\tau_{i,t} - 2\tau_{i,t-1} + \tau_{i,t-2} \to 0$, so that every element of $\boldsymbol{\tau}_t$ tends to behave like a straight line. Now, the amount of smoothness of a trend depends only on its length (N), the value of the smoothing parameter (λ) and the correlation between cycles, as it can be seen in the smoothness index (A10) presented below. Thus, we should notice that only one smoothing constant is used to smooth both trends involved since in the present situation both time series: (i) have the same length and (ii) share the same amount of smoothness.

Alternatively, if we consider the stacked vectors $\mathbf{Z} = (\mathbf{Z}_1',\ldots,\mathbf{Z}_N')'$ and $\boldsymbol{\tau} = (\boldsymbol{\tau}_1',\ldots,\boldsymbol{\tau}_N')'$, the problem can be posed as that of minimizing

$$M(\lambda) = (\mathbf{Z} - \boldsymbol{\tau})'(I_N \otimes W)(\mathbf{Z} - \boldsymbol{\tau}) + \lambda\boldsymbol{\tau}'(K'K \otimes I_2)\boldsymbol{\tau} \tag{A3}$$

where I_N is the N-dimensional identity matrix, $\otimes$ denotes Kronecker product and K is an (N-2)×N matrix representing the second order difference operator, that is,

$$K=\begin{pmatrix} 1 & -2 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{pmatrix}. \tag{A4}$$

The solution to this problem is provided by Guerrero *et al*. (2016), that is

$$\hat{\boldsymbol{\tau}} = (I_{2N} + \lambda K'K \otimes W^{-1})^{-1}\mathbf{Z}. \tag{A5}$$

It should be stressed that (A5) is valid when both λ and W are known. Thus, the practical problem lies in providing adequate values for those parameters. GIR solved such a problem by applying Generalized Least Squares (GLS) on the assumption that the smooth trend behavior is described by means of

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\tau}_t - 2\boldsymbol{\tau}_{t-1} + \boldsymbol{\tau}_{t-2} \text{ for } t = 3, 4, \ldots, N. \tag{A6}$$

and making $W = \Sigma_\eta^{-1}$ and $\lambda I_2 = \Sigma_\varepsilon^{-1}$, with Σ_η the symmetric variance-covariance matrix of $\boldsymbol{\eta}_t$ and Σ_ε the diagonal variance-covariance matrix of $\boldsymbol{\varepsilon}_t$, that is

$$\Sigma_\eta = \begin{pmatrix} \sigma^2_{\eta_1} & \sigma_{\eta_1\eta_2} \\ \sigma_{\eta_2\eta_1} & \sigma^2_{\eta_2} \end{pmatrix} \text{ and } \Sigma_\varepsilon = \begin{pmatrix} \sigma^2_{\varepsilon_1} & 0 \\ 0 & \sigma^2_{\varepsilon_2} \end{pmatrix}. \tag{A7}$$

Then the variance-covariance matrix of the GLS estimate is given by

$$\mathrm{Var}(\hat{\boldsymbol{\tau}}) = (I_2 \otimes \Sigma_\eta^{-1} + K'K \otimes \Sigma_\varepsilon^{-1})^{-1} \tag{A8}$$

where Σ_η and Σ_ε are assumed known. It should be stressed that GIR showed that the correlation between trends that may be taken into account by using a non-diagonal matrix Σ_ε is practically irrelevant for trend estimation. In fact, they showed that even correlations as high as

0.95 produce an increment in smoothness from a desired 80% to something slightly over 83% for sample sizes between 100 and 400.

A.2 Smoothing constant chosen by controlling smoothness

A feasible solution to the above problem is based on the main idea that the smoothing constant should be calibrated not estimated in order to avoid the need of validating the assumptions of the model underlying the estimation method employed. Thus, we suggest choosing the value of λ in such a way as to provide estimated trends with a percentage of smoothness chosen by the analyst on a priori grounds. The percentage of smoothness is related to the precision (the inverse of the variance-covariance matrix) of the estimated trend as follows. First, let us notice that the precision of the trend estimate is given by the sum of two precisions, one for the unobserved-component model (A1) and the other for the smoothness representation (A6), that is,

$$\Gamma^{-1} = [\mathrm{Var}(\hat{\boldsymbol{\tau}})]^{-1} = \mathrm{I}_2 \otimes \Sigma_\eta^{-1} + \mathrm{K'K} \otimes \Sigma_\varepsilon^{-1}. \qquad \text{(A9)}$$

Thus, the proportion of precision attributable to smoothness equals the proportion of the matrix $\mathrm{K'K} \otimes \Sigma_\varepsilon^{-1}$ with respect to Γ^{-1}. A scalar measure of the proportion of P in P+Q, for P and Q two symmetric positive definite matrices is given by $\Delta(\mathrm{P};\mathrm{P}+\mathrm{Q}) = \mathrm{tr}[\mathrm{P}(\mathrm{P}+\mathrm{Q})^{-1}]/\mathrm{N}$. This is the only measure satisfying: (i) it takes on values in (0; 1); (ii) it adds up to one, in the sense that $\Delta(\mathrm{P};\mathrm{P}+\mathrm{Q}) + \Delta(\mathrm{Q};\mathrm{P}+\mathrm{Q}) = 1$; (iii) it is invariant under linear non-singular transformations of the variable involved, and (iv) it behaves linearly. See Theil (1963) for a proof. When the two matrices involved are precision matrices we refer to $\Delta(P;P+Q)$ as a measure of precision share of P in P+Q.

The measure of precision share is considered a smoothness index in GIR where it is expressed as

$$\Delta(K'K \otimes \Sigma_{\varepsilon}^{-1};\Gamma^{-1}) = 1 - tr[(I_{2N} + \lambda\beta \otimes K'K)^{-1}]/(2N) \qquad \text{(A10)}$$

with $\beta = \Sigma_{\eta}\mathrm{Diag}(\sigma_{\eta_1}^{-2}, \sigma_{\eta_2}^{-2})$ the matrix of regression coefficients of one cycle on the other. It should also be noticed that no correlation is allowed between the two trends involved, which is not too restrictive, as shown by GIR. They found that only some negligible amount of over-smoothness occurs if we ignore such a correlation. The importance of the smoothness index lies in that we can fix at the outset of the study some desired percentage of smoothness to be achieved by the trend estimate and solve equation (A10) for λ, given a preliminary estimate of the matrix β. As an aid to choose an appropriate percentage of smoothness, GIR provided some guidelines that arise from a simulation study, and indicate how to obtain the required preliminary estimates. The following section presents the basic steps required to apply GIR's procedure, as well as an extract of the most important table.

A.3 Summary of GIR's procedure

We present here a brief description of GIR's algorithm and the most important table used to estimate bivariate trends.

Algorithm

a) Compute preliminary estimates of Σ_{η} and Σ_{ε}.

b) Let the data suggest a value for the smoothing parameter and preliminarily estimate the trends and the correlation between cycles.

c) With the values of N and the correlation between η_y and η_u, say $\tilde{\rho}_{\eta_y \eta_u}$, choose λ from Table 3 (see the extract below) so that a pre-specified percentage of trend smoothness can be achieved.

d) Compute the final estimates of the bivariate trend $\hat{\boldsymbol{\tau}}$ as in (A5) and $\mathrm{Var}(\hat{\boldsymbol{\tau}})$ as in (A8).

Extract of Table 3 in GIR

Table A.1. Smoothing parameter λ for some choices of sample size (N) and between-cycle correlation (ρ) for 80% smoothness

N / ρ	…	0.4	0.6	0.8	…
224	…	13.62	16.29	24.05	…
256	…	13.48	16.12	23.81	…

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Figure 1. Mexican cyclical output and unemployment rate estimates: Bivariate HP filter with 90% of smoothness, 1993:QI-2015:QII.

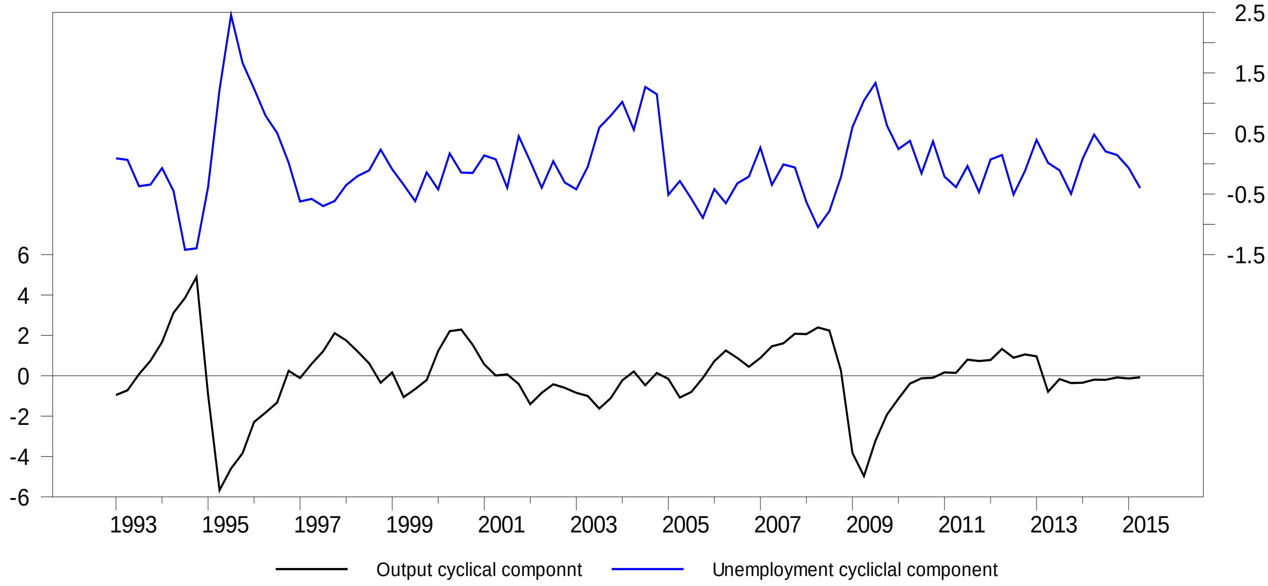

Figure 2. Smoothed probabilities of being in a recessionary regime (TVTP)

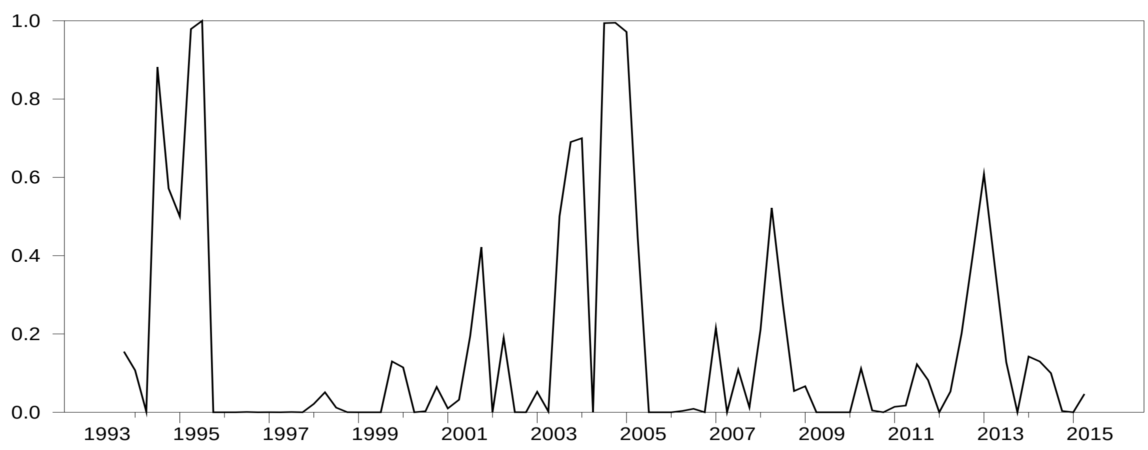

Table 1. Estimated results from models I, II and III

Model I		Model II		Model III	
Parameter	Estimated	Parameter	Estimated	Parameter	Estimated
α_0	-0.0010 (0.0430)	α_{01}	-0.0374 (0.0395)	α_{01}	-0.0567** (0.0288)
β_1	-0.1806*** (0.0314)	β_1	-0.1207*** (0.0377)	β_1	-0.1092*** (0.0295)
α_1	0.4404*** (0.0623)	α_{11}	0.4059*** (0.0924)	α_{11}	0.3875*** (0.0220)
		α_{21}	0.0335 (0.186)	α_{21}	-0.0601 (0.0106)
		α_{31}	0.0894 (0.1025)	α_{31}	0.2070** (0.0971)
		α_{41}	-0.2414** (0.0876)	α_{41}	-0.3751*** (0.1009)
		α_{51}	0.0002 (0.0832)	α_{51}	0.1523 (0.1119)
		α_{02}	0.2259** (0.0923)	α_{61}	-0.1543* (0.0828)
		β_2	-0.3117*** (0.0152)	α_{20}	0.2307* (0.0932)
		α_{12}	0.7537*** (0.1391)	β_2	-0.2606*** (0.0369)
		α_{22}	-0.4835* (0.2534)	α_{21}	0.5920*** (0.1572)
		α_{32}	1.2359*** (0.2754)	α_{22}	0.0803 (0.2241)
		α_{42}	-0.7215** (0.2417)	α_{32}	0.0503 (0.1992)
		α_{52}	-0.9283*** (0.2742)	α_{42}	0.1812 (0.2021)
		σ_{11}^2	0.0811** (0.0166)	α_{52}	-0.3859* (0.2224)
		σ_{12}^2	0.0445** (0.0219)	α_{62}	-0.0918 (0.1474)
		p_{11}	0.9054*** (0.0650)	σ_{11}^2	0.0813*** (0.0155)
		p_{12}	0.4650* (0.2814)	σ_{12}^2	0.0756** (0.0242)
				δ_{10}	2.8524** (0.5551)
				δ_{11}	2.4859* (1.4212)
				δ_{20}	3.3282** (1.5971)
				δ_{21}	-4.7561* (2.8266)
Log L -40.0831		*Log L -26.5087*		*Log L -22.3847*	
Model selection test: Likelihood ratio test					
Model I vs Model II: 27.148***			Model II: vs Model III: 8.2574*		
Model II specification test: $H_0^{ss}: \alpha_{j1} = \alpha_{j2}, \beta_1 = \beta_2, ; j = 0,1,2, \ldots, 5$: 62.372***					
Model III specification test: $H_0^{ss}: \alpha_{j1} = \alpha_{j2}, \beta_1 = \beta_2, ; j = 0,1,2, \ldots, 6$: 44.774***					
Test of asymmetry: Model II $H_0: \beta_1 = \beta_2$ 15.923*** Model III $H_0: \beta_1 = \beta_2$ 11.952***					

Notes: Numbers in brackets are SEs. ***, **,* denotes significance at the 1%,5% and 10%level, respectively.