THE IMPACTS OF UNEMPLOYMENT ON MEXICO’S OUTPUT GAP, 1993*Q*1-2016*Q*4

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Abstract

Using quarterly data for Mexico from 1993*Q*1 to 2016*Q*4, we measure the impacts of unemployment on Mexico’s output gap. The main purpose and contribution to the Mexican literature of our paper are to measure the relationship between output and unemployment by estimating two time-varying coefficients (TVC) models. From an econometric modeling point of view, these models allow asymmetric interactions between output and unemployment. Our principal conclusion: Okun’s law is a strong and stable relationship in Mexico. When actual Gross Domestic Product is less than potential output, the unemployment gap rises by 0.17 percentage points. It implies that an increment of unemployment gap of one percent is associated with an output loss of -5.88 of potential output.

JEL Classification: C13, C22, E32

Keywords: equilibrium unemployment rate, Kalman filter, NAIRU, natural unemployment rate, Okun’s law, output gap, output growth, potential output, state-space models, time-varying coefficients, unemployment

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1. Introduction

The difference between the actual Gross Domestic Product (GDP) of an economy and its maximum sustainable output consistent over the medium-term with a stable inflation rate (potential output) is known as *output gap*. The output gap can be viewed as both a measure of economic fluctuations and as an indicator of economic efficiency: On the one hand, a positive output gap is typically accompanied by rising inflation because actual output is higher than economy's maximum-efficiency output. On the other hand, a negative output gap is usually accompanied by falling inflation because actual output is below the economy's full capacity. In his seminal work, Arthur Okun (1962) reported a negative short-run relationship between the unemployment rate and the output gap, which became known as *Okun's law*. This empirical relationship can be written as:

$$u_t - \tilde{u}_t = \mu - \alpha\left(y_t - \tilde{y}_t\right) \tag{1}$$

where u_t = unemployment rate; $\tilde{u}_t$ = equilibrium unemployment rate; y_t = logarithm of actual GDP; $\tilde{y}_t$ = logarithm of potential output; and μ is the expected unemployment gap.

From expression (1) or *gaps model*, we can deduce that if actual GDP is equal to the potential output, then actual unemployment rate equals the equilibrium unemployment rate (in other words, there is "full employment"). Implicit in this is the notion that there is only one level of unemployment that is consistent with a zero output gap; that is to say, there is one level of *full-employment* GDP.

The equilibrium unemployment rate is related to two theoretical concepts in modern macroeconomics: The first one is the *natural rate of unemployment;* it appeared in Friedman (1968, p.8):

> "The natural rate of unemployment", in other words, is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of mobility, and so on.

The second concept is the *non-accelerating rate of unemployment* (NAIRU). NAIRU is the rate of unemployment consistent with a stable rate of inflation (Tobin; 1980; Modigliani and Paoademos, 1975), i.e. the rate of unemployment at which inflation will neither rise nor decrease.

The natural rate of unemployment and NAIRU are often viewed as two names of the same economic idea (Staiger et al., 1997; Gordon, 1997). However, Estrella and Mishkin (1998) argue that the natural rate of unemployment is not the same thing as the NAIRU. On the one hand, the natural rate of unemployment is the sum of the frictional and structural unemployment rates: it changes slowly over time because is the unemployment rate that the economy reaches in the long run. On the other hand, NAIRU is related to past levels of unemployment and inflation: it fluctuates much more than the natural rate of unemployment. In this work we use equilibrium unemployment rate, the natural rate of unemployment and NAIRU as synonymous because both concepts can be understood as reflections of the output gap (Posta, 2008): when the economy is at the natural rate of unemployment, the inflation rate is constant and there is macroeconomic equilibrium.

In practice, the equilibrium unemployment rate and the potential output are unobserved, but they can be measured. However, if we assume that potential output grows at a constant rate, $\Delta\tilde{y}_t = 0$, and the equilibrium unemployment rate is constant over the medium-term, $\Delta\tilde{u}_t = 0$, the first difference of gaps model is:

$$\Delta u_t = -\beta \Delta y_t \qquad (2)$$

From expression (2) or *first-difference model*, we can measure the impact of output growth on changes in unemployment rate. While in this model unemployment and output are expressed in percentage points, in the gaps model they are measured in terms of deviations from long-term trends (i.e. cyclical components).

From expressions (1) and (2) we can measure the impacts of unemployment on output or *Okun's law coefficients*: $-(1/\alpha)$ and $-(1/\beta)$, respectively. However, both coefficients assume that unemployment has the same absolute effect on output over time; in other words, Oku's law is symmetric. Courtney (1991) and Palley (1993) are among the initial contributors to the idea that the relationship between output and unemployment is nonlinear. Courtney concluded that imposing symmetry on Okun's law leads to "serious underestimates of unemployment rate increases in contractions and overestimates of decreases in the unemployment rate during expansions". Palley claims that Okun's law has become more cyclically sensitive. Viren (2001) says that asymmetry would provide an explanation for the varying effectiveness of unemployment policies. Harris and Silverstone (2001) argue that ignoring asymmetry in Okun's law when it is present leads to misspecified econometric models and faulty policy conclusions.

The main purpose and contribution to the Mexican literature of our paper are to measure the relationship between output and unemployment by estimating two time-varying coefficients (TVC) models. From an econometric modeling point of view, these models allow asymmetric interactions between output and unemployment. Therefore, this paper contributes to the literature on asymmetry in Okun's law using quarterly data for Mexico from 1993*Q*1 to 2016*Q*4.

This paper is organized as follows: Section 2 presents our econometric approach of Okun's law; section 3 shows the statistical relationship between unemployment and output gaps of Mexico, and measures the impacts of unemployment on Mexico's output gap. Finally, conclusions are in section 4.

2. Methodology

In this study, two time-varying coefficients (TVC) models are adopted to estimate the impacts of unemployment on Mexico's output growth. Econometric inference always imposes some model assumptions, linearity being among the most important. Although linear models are useful, they are often unrealistic in economic applications (Durbin and Koopman, 2001); moreover, misspecification of Data Generating Mechanism by a linear model could lead to large bias (Fan and Zhang, 2008). Many studies in the current international literature tend to pay more attention to the possibility of asymmetry in Okun's coefficient (Lancaster and Tulip, 2015; Cheng et al. 2015; Silvapulle et al., 2004; Harris and Silverstone, 2001; Lee, 2000).

2.1. Baseline models

Based on expressions (1) and (2), the baseline models are:

$$g_t^u = \mu - \alpha g_t^y + \varepsilon_t \qquad (3)$$

$$\Delta u_t = -\beta \Delta y_t + \widehat{\varepsilon}_t \qquad (4)$$

where $g_t^u = u_t - \tilde{u}_t$; $g_t^y = y_t - \tilde{y}_t$; $\varepsilon_t \sim niid\left(0, \sigma_\varepsilon^2\right)$; $\hat{\varepsilon}_t \sim niid\left(0, \sigma_{\hat{\varepsilon}}^2\right)$; the noise processes are assumed to be uncorrelated.

Models (3) and (4) show, respectively, the effects of output gap on unemployment gap, the effects of changes in output gap on changes unemployment gap compared to previous quarter.

On the one hand, the impacts of output on unemployment are: $-\alpha$ and $-\beta$. On the other hand, the linear impacts of unemployment on output or *symmetric* Okun's law coefficients are: $-(1/\alpha)$ and $-(1/\beta)$. However, we anticipated misspecification problems in models (3) and (4) because of its simplicity. Misspecification is a fundamental problem that can lead to inconsistent coefficient estimates and incorrect interpretations.

2.2. Autoregressive distributed lag models

Additionally, we will estimate the following autoregressive distributed lag (ARDL) models:

$$\delta(L) g_t^u = \mu - \alpha(L) g_t^y + \zeta_t \tag{5}$$

$$\theta(L) \Delta g_t^u = -\beta(L) \Delta g_t^y + \hat{\zeta}_t \tag{6}$$

where $\delta(L)$, $\alpha(L)$, $\theta(L)$, and $\beta(L)$ are 6-order polynomials; $\zeta_t \sim niid\left(0, \sigma_\zeta^2\right)$; $\hat{\zeta}_t \sim niid\left(0, \sigma_{\hat{\zeta}}^2\right)$; the noise processes are assumed to be uncorrelated.

Models (5) and (6) show, respectively, the dynamic marginal effects of output gap on unemployment gap, the dynamic marginal effects of changes in output gap on changes unemployment gap compared to previous quarter.

On the one hand, the long-run impacts of output on unemployment are: $-[\alpha(L)/\delta(L)]$ and $-[\beta(L)/\theta(L)]$. On the other hand, the symmetric Okun's law coefficients are: $-[\delta(L)/\alpha(L)]$ and $-[\theta(L)/\beta(L)]$.

The goal of ARDL models is to reflect, concisely and accurately, important dynamic interactions (temporary and permanent changes) over the long-run between unemployment and output gaps. However, from an econometric point of view, estimation of models (5) and (6) presents the following challenges: First, all of the variables should be stationary. Second, the problem of measuring potential output and equilibrium unemployment rate. Third, if ζ_t and $\hat{\zeta}_t$ are not white noises, then $\sum_{i=1}^{6} g_{t-i}^{u}$ and $\sum_{j=1}^{6} \Delta g_{t-j}^{u}$ are not strictly exogenous and the OLS estimators are inconsistent. Finally, economic theory does not give us any information about the chosen lag-length in expressions (5) and (6). Consequently, we are forced to make a judgment after running the regressions.

2.3. Measuring potential output and equilibrium unemployment rate

Potential output and equilibrium unemployment rate play a key role in all models. In practice, they are not directly observed, but they can be measured (International Monetary Fund; 2015; Cui et al., 2015; Arnold, 2009; Rodenburg, 2007; Congressional Budget Office, 2004; Weiner, 1993). There are many techniques to estimate potential output and equilibrium unemployment rate, such as Hodrick-Prescott filter, Baxter-King Band-Pass filter, Structural Vector Autoregression Approach, and Production Function Approach. State-space models are particularly useful for models involving unobserved or *hidden* variables. They have been used extensively in system theory, the physical sciences, engineering, and economics.

In a state-space model (Durbin and Koopman, 2001), we have an equation for determining the unknown *state* of the system which is driven by a stochastic process, and an equation for determining the observed *signal* of the system. The *Kalman filter* (Kalman, 1940; Kalman and Bucy, 1961) is an algorithm for *prediction* and *updating* equations for determining the optimal estimates of the state equations given information available, i.e. given the observable signal, the Kalman filter provides estimates of the states signals and measures of the precision of these estimates.

The following state-space models of potential output and equilibrium unemployment rate can be estimated by the Kalman filter, equations (7) and (9) are the signal equations, and equations (8), and (10) are the state equations:

$$y_t = \tilde{y}_t + \varepsilon_t^y \tag{7}$$

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \phi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \phi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^\phi \end{bmatrix} \tag{8}$$

$$u_t = \tilde{u}_t + \varepsilon_t^u \tag{9}$$

$$\begin{bmatrix} \tilde{u}_{t+1} \\ \varphi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{u}_t \\ \varphi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^\varphi \end{bmatrix} \tag{10}$$

where $\varepsilon_t^y \sim niid\left(0, \sigma_{\varepsilon^y}^2\right)$; and $\varepsilon_t^\phi \sim niid\left(0, \sigma_{\varepsilon^\phi}^2\right)$, $\varepsilon_t^u \sim niid\left(0, \sigma_{\varepsilon^u}^2\right)$; and $\varepsilon_t^\varphi \sim niid\left(0, \sigma_{\varepsilon^\varphi}^2\right)$; the noise processes are assumed to be uncorrelated.

State-space models are more flexible than other methods. While linear regression models use exogenous variables to distinguish the explained variation from the unexplained variation, state-space models depend on the dynamics of the state variables and the linkage between the observed variables and state variables to draw statistical inference about the unobserved states. Under normality assumptions, the estimator of the state produced by the

Kalman filter is the conditional expectations $E(\tilde{y}_t \mid y_1,...,y_T)$ and $E(\tilde{u}_t \mid u_1,...,u_T)$, also provides the conditional covariance matrixes $\text{cov}(\tilde{y}_t \mid y_1,...,y_T)$ and $\text{cov}(\tilde{u}_t \mid u_1,...,u_T)$. The computation of the estimators $E(\tilde{y}_t \mid y_1,...,y_T)$ and $E(\tilde{u}_t \mid u_1,...,u_T)$ is called *filtering*.

However, our estimates of potential output and equilibrium unemployment rate have limitations: First, such estimates are purely statistical approximations of theoretical concepts and thus contain an element of randomness. Second, our results can be interpreted, on the one hand, as *trend* output but not as *potential* output; on the other hand, as *trend* unemployment rate but not as *equilibrium* unemployment rate. Meta-analysis of existing literature goes beyond our purview.

2.4. Time-varying coefficients models

The meaning of "asymmetry" is that the reaction of unemployment to output is different when the output is increasing from that when the output is decreasing, i.e. output does not have the same absolute effect on unemployment over time. Although models (3)-(6) let us interpret all the estimated coefficients, the assumption of linearity (i.e. there is a long-run relationship between unemployment and output gaps) can be too restrictive. Furthermore, from an economic point of view, models (3)-(6) are subject to the *Lucas critique* (Lucas, 1976) and fails to take into account the inherent nonlinearities in Oku's law.

A TVC models represent a generalization of the classic linear models in which the coefficients are allowed to change over time; they relax stationarity assumptions, and also provide a simple interpretation of estimated coefficients. The following TVC models are useful for exploring nonlinear interactions (i.e. time-varying effects) between unemployment and output gaps:

$$g_t^u = \mu_t - (\alpha_t - a) g_t^y + \xi_t \tag{11}$$

$$\begin{bmatrix} \mu_{t+1} \\ (\alpha_{t+1} - a) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \theta \end{bmatrix} \begin{bmatrix} \mu_t \\ (\alpha_t - a) \end{bmatrix} + \begin{bmatrix} \upsilon_t \\ \nu_t \end{bmatrix} \tag{12}$$

$$\Delta g_t^u = \omega_t - (\beta_t - b) \Delta g_t^y + \hat{\xi}_t \tag{13}$$

$$\begin{bmatrix} \omega_{t+1} \\ (\beta_{t+1} - b) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \hat{\theta} \end{bmatrix} \begin{bmatrix} \omega_t \\ (\beta_t - b) \end{bmatrix} + \begin{bmatrix} \hat{\upsilon}_t \\ \hat{\nu}_t \end{bmatrix} \tag{14}$$

where a and b are constants; $\xi_t \sim niid\left(0, \sigma_{\xi}^2\right)$; $\hat{\xi}_t \sim niid\left(0, \sigma_{\hat{\xi}}^2\right)$; $\upsilon_t \sim niid\left(0, \sigma_{\upsilon}^2\right)$; $\hat{\upsilon}_t \sim niid\left(0, \sigma_{\hat{\upsilon}}^2\right)$; $\nu_t \sim niid\left(0, \sigma_{\nu}^2\right)$; $\hat{\nu}_t \sim niid\left(0, \sigma_{\hat{\nu}}^2\right)$; the noise processes are assumed to be uncorrelated.

Models (11)-(14) show, respectively, the asymmetric responses of unemployment gap to output gap, the nonlinear effects of changes in output gap on changes unemployment gap compared to previous quarter, and the nonlinear effects of changes in output gap on changes in unemployment gap compared to the same quarter of previous year.

On the one hand, the elements μ_t and ω_t captures unobserved effects between unemployment and output gaps. On the other hand, the time-varying coefficient capture both permanent and temporary changes in a flexible and robust manner: When θ and $\hat{\theta}$ are equal to zero, the state dynamics are given by $\alpha_t = a + \nu_t$ and $\beta_t = b + \hat{\nu}_t$, respectively; if σ_{ν}^2 and $\sigma_{\hat{\nu}}^2$ are small relative to *a* and *b*, respectively, the systems are nearly deterministic; i.e. $\alpha_t \approx a$ and $\beta_t \approx b$. Therefore, the *asymmetric* Okun's law coefficients are: $-\left[1/(\alpha_t - a)\right]$ and $-\left[1/(\beta - b)\right]$.

3. Results

In this section, we estimate models (3)-(14). The data series used in this research, GDP and Harmonised Unemployment Rate (HUR), are quarterly data for Mexico covering the period 1993*Q*1-2016*Q*4 (OECD, 2017a and 2017b). This paper contributes, alongside several economists, to the Mexican literature on asymmetry in Okun's law; the list includes, inte alia, Loría, et al. (2015), Islas-Camargo and Cortez (2013), Loría, et al. (2012), De Jesús and Carbajal (2011), Islas-Camargo and Cortez (2011), Loría and de Jesús (2011), Loría and García-Ramos (2008), Rodríguez and de Jesús (2007), Liquitaya and Lizarazu (2004), and Chavarín (2001). However, through models (11)-(14), this paper allows nonlinearities in Okun's law and provides an updating of Okun's law coefficients.

3.1. The statistical relationship between unemployment and output gaps of Mexico

Figures 1 and 2 plot, respectively, the evolution of unemployment and output gaps from 1993*Q*1 to 2016*Q*4, and changes in unemployment and output gaps compared to previous quarter from 1993*Q*2 to 2016*Q*4. All variables are stationary (see Table 1A of the appendix). The dates indicated in all figures show the two major economic contractions. There is a common pattern: A positive unemployment gap is related to a negative output gap; conversely, a negative unemployment gap is related to a positive output gap.

Figure 1. Mexico: evolution of g^u and g^y, 1993Q1-2016Q4

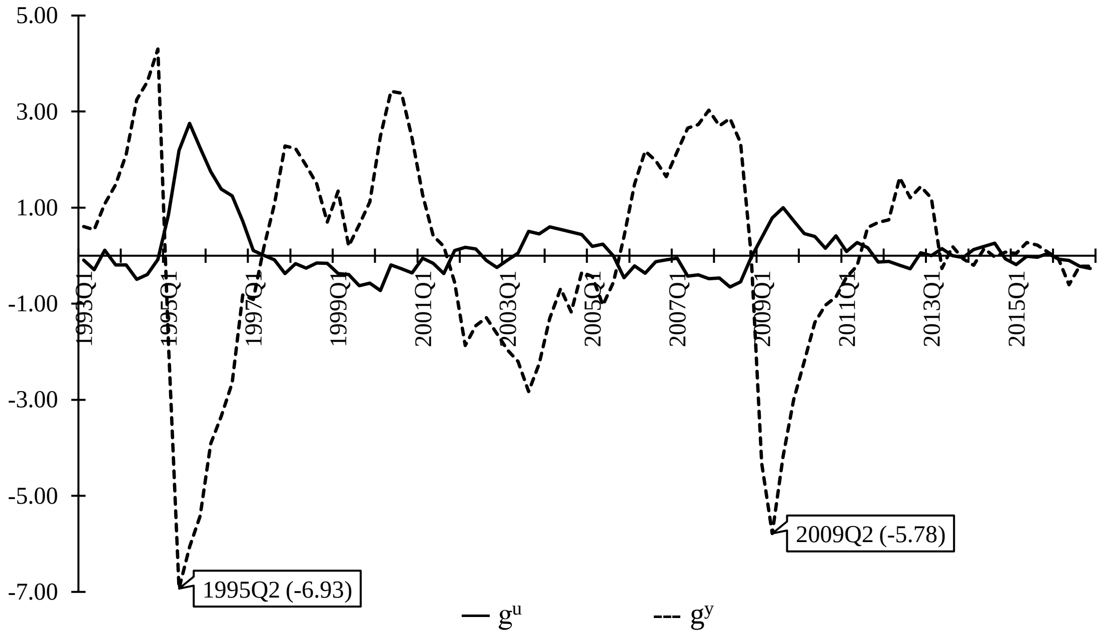
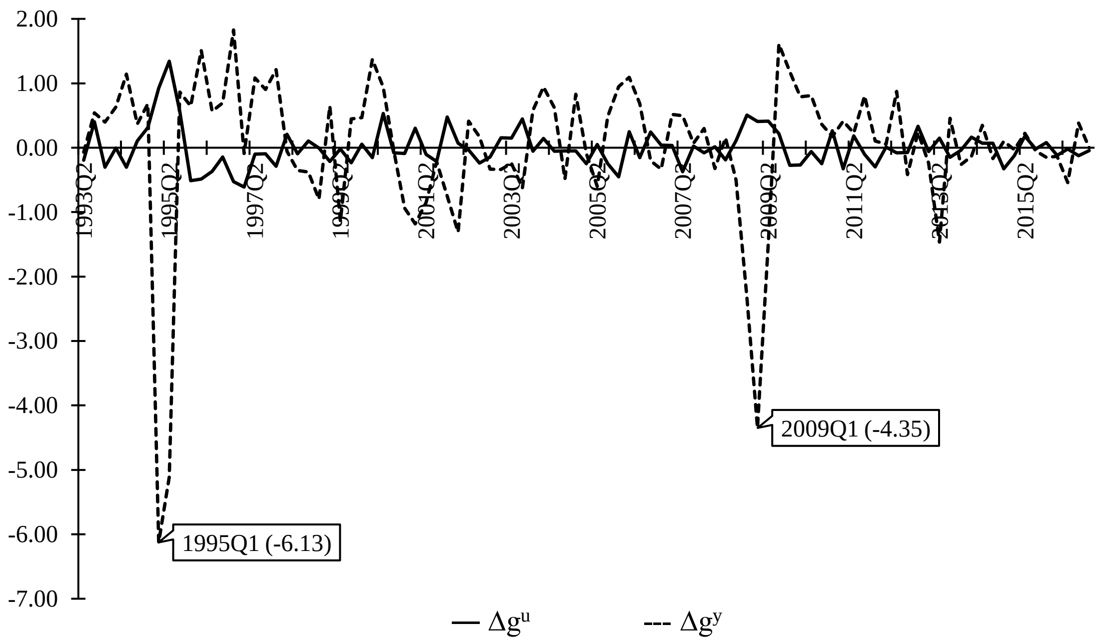


Figure 2. Mexico: evolution of Δg^u and Δg^y, 1993Q2-2016Q4



Figures 3 and 4 present, respectively, the cross-correlograms between unemployment and output gaps, and between changes in unemployment and output gaps compared to previous quarter. All correlations are asymptotically consistent approximations; the dotted lines in the cross-correlograms are the approximate two standard error bounds. All figures provide a quantitative assessment of the likeness of unemployment and output gaps at all possible statistically significant lags. They show the asymmetrical cyclical relationship of the variables: positive correlation values indicate that as one variable rises so does the other, and negative correlation values indicate that as one variable rises the other decreases. The lags indicated in all figures show maximum correlations.

Figure 3. Cross-correlogram of g^u and g^y

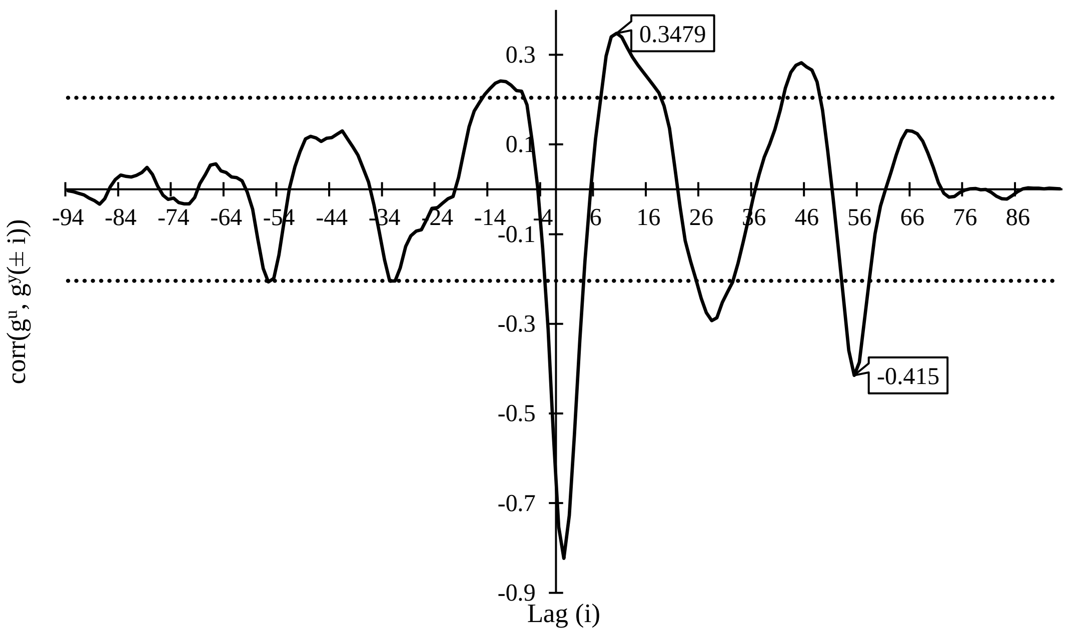
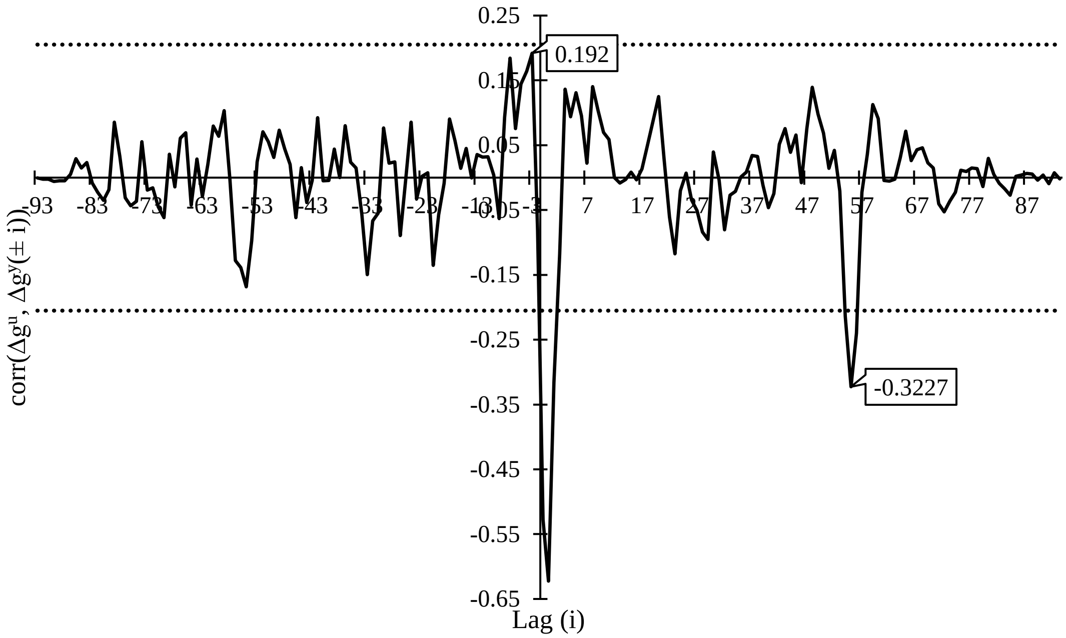


Figure 4. Cross-correlogram of Δg^u and Δg^y



Figures 5 and 6 are, respectively, the scatterplots of unemployment and output gaps from 1993*Q*1 to 2016*Q*4, and changes in unemployment and output gaps compared to previous quarter from 1993*Q*2 to 2016*Q*4. All figures display a linear regression line (dashed line) and a polynomial regression line (dotted line). The R^2 of the linear regressions are 0.68 and 0.39, respectively. The R^2 of the polynomial regressions are 0.76 and 0.45, respectively. Therefore, the polynomial regression lines can be considered acceptable estimations of the "true" relationship between unemployment and output gaps.

Figure 5. Scatterplot of g^u and g^y, 1993*Q*1-2016*Q*4

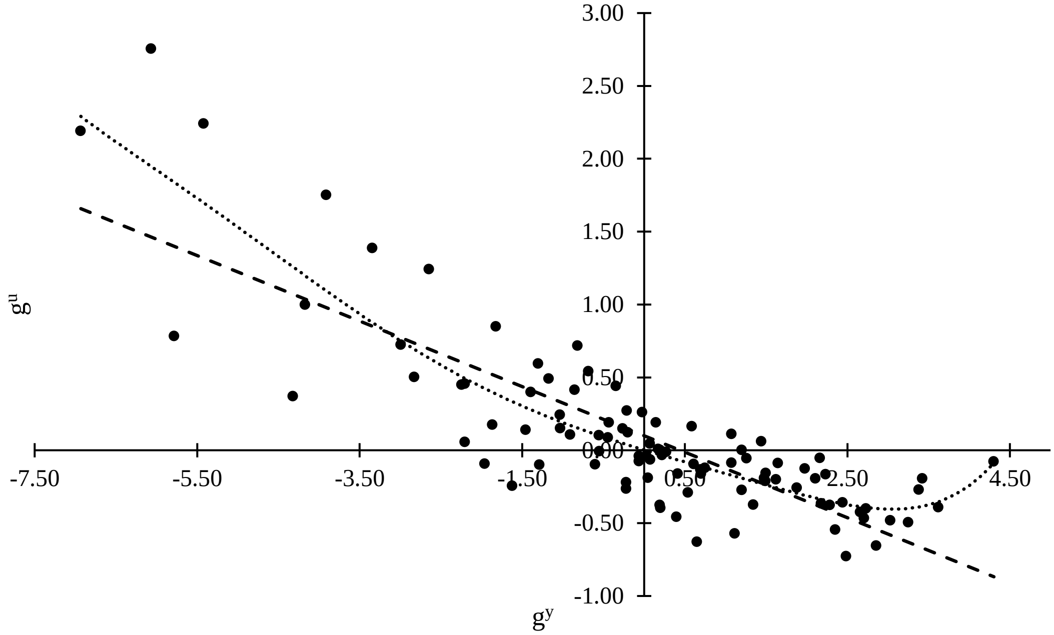
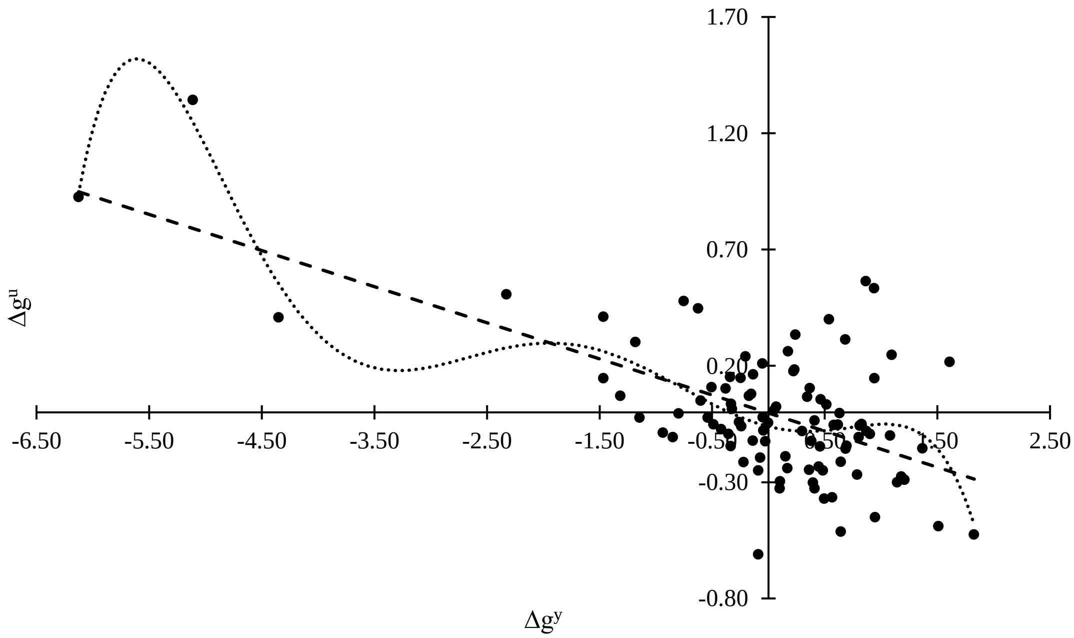


Figure 6. Scatterplot of Δg^u and Δg^y, 1993*Q*2-2016*Q*4



Certainly, correlation does not necessarily imply causation. Table 1 presents the *pairwise Granger causality tests* for unemployment and output gaps. We cannot reject the hypothesis that g_t^y and Δg_t^y *Granger* cause g_t^u and g_t^u, respectively. Therefore, it seems that *Granger* causality runs one-way from output gap to unemployment gap and not the other way.

Table 1. Pairwise Granger causality tests

	Lags			
	1-2	1-4	1-6	1-8
	F-Statistic (Probability)			
g_t^y does not *Granger* cause g_t^u	12.07 (0.00)	5.57 (0.00)	3.44 (0.00)	2.76 (0.01)
g_t^u does not *Granger* cause g_t^y	2.00 (0.14)	1.76 (0.15)	1.36 (0.24)	1.27 (0.27)
Δg_t^y does not *Granger* cause Δg_t^u	12.95 (0.00)	5.72 (0.00)	3.64 (0.00)	4.33 (0.00)
Δg_t^u does not *Granger* cause Δg_t^y	3.07 (0.05)	2.22 (0.07)	1.80 (0.11)	0.87 (0.54)

3.2. The impacts of unemployment on Mexico's output gap

This section presents the econometric findings. All coefficients are statistically significant; Tables 2A-7A report all goodness-of-fit and diagnostic statistics. Adjusted R-squared and standard error in parentheses are reported.

3.2.1. Symmetric Okun's law coefficients

On the one hand, expressions (15) and (16) show the estimated coefficients of baseline models, respectively. Model (16) is the best one, the other have misspecification problems.

$$\underset{R^2=0.68}{g_t^u} = \underset{(0.03)}{0.10} - \underset{(0.02)}{0.22 g_t^u} \tag{15}$$

$$\underset{R^2=0.50}{\Delta g_t^u} = -\underset{(0.02)}{0.17 \Delta g_t^y} + \underset{(0.21)}{0.70 d_{95Q3}} + \underset{(0.21)}{0.69 d_{00Q2}} \tag{16}$$

where $d_{95Q3} = \begin{cases} 1 \text{ if } t = 1995Q3 \\ 0, \text{ otherwise} \end{cases}$ and $d_{00Q2} = \begin{cases} 1 \text{ if } t = 2000Q2 \\ 0, \text{ otherwise} \end{cases}$.

From models (15) and (16), the effects of output gap on unemployment gap are: -0.22 and -0.17, respectively; e.g. from expression (15), when actual GDP is greater than potential output, the unemployment gap falls by 0.22 percentage points. It implies that an increment of unemployment gap of one percent is associated with an output loss of -4.45 of potential output. The symmetric Okun's law coefficients are: -4.45 and -6.06, respectively.

On the other hand, expressions (17) and (18) show the estimated coefficients of ARDL models, respectively; by convention in dynamic analysis, we include all lags even if its coefficients are not statistically significant.

$$\underset{R^2=0.90}{g_t^u} = \underset{(0.06)}{0.84 g_{t-1}^u} - \underset{(0.02)}{0.12 g_t^y} + \underset{(0.03)}{0.01 g_{t-1}^y} + \underset{(0.02)}{0.09 g_{t-2}^y} + \underset{(0.20)}{0.72 d_{00Q2}} \tag{17}$$

$$\underset{R^2=0.55}{\Delta g_t^u} = -\underset{(0.09)}{0.06 \Delta g_{t-1}^u} - \underset{(0.02)}{0.13 \Delta g_t^y} - \underset{(0.02)}{0.10 \Delta g_{t-1}^y} + \underset{(0.21)}{0.77 d_{00Q2}} \tag{18}$$

where $d_{00Q2} = \begin{cases} 1 \text{ if } t = 2000Q2 \\ 0, \text{ otherwise} \end{cases}$.

From models (17) and (18), the long-run effects of output gap on unemployment gap are: -0.16 and -0.21, respectively; e.g. from expression (17), when actual GDP is greater than potential output, the unemployment gap falls by 0.16 percentage points. It implies that an increment of unemployment gap of one percent is associated with an output loss of -6.23 of potential output. The symmetric Okun's law coefficients are: -6.23 and -4.75, respectively.

3.2.2. Asymmetric Okun's law coefficients

Expressions (19) and (20) report the estimated coefficients of TVC models at final state (2016Q4):

$$\underset{R^2=0.90}{g_t^u} = -0.33\mu_t - \underset{(0.02)}{0.17 g_t^y} + \underset{(0.15)}{0.62 d_{95Q3}} - \underset{(0.20)}{0.65 d_{97Q1}} + \underset{(0.20)}{0.69 d_{00Q2}} \tag{19}$$

$$\underset{R^2=0.57}{\Delta g_t^u} = -0.01\omega_t - \underset{(0.02)}{0.17}\Delta g_t^y + \underset{(0.20)}{0.72}d_{95Q3} - \underset{(0.20)}{0.61}d_{97Q1} + \underset{(0.20)}{0.70}d_{00Q2} \qquad (20)$$

where $d_{95Q3} = \begin{cases} 1 \text{ if } t = 1995Q3 \\ 0, \text{ otherwise} \end{cases}$, $d_{97Q1} = \begin{cases} 1 \text{ if } t = 1997Q1 \\ 0, \text{ otherwise} \end{cases}$, $d_{00Q2} = \begin{cases} 1 \text{ if } t = 2000Q2 \\ 0, \text{ otherwise} \end{cases}$, and

$d_{01Q2} = \begin{cases} 1 \text{ if } t = 2001Q2 \\ 0, \text{ otherwise} \end{cases}$.

In all models, the asymmetric long-run response of unemployment gap to output gap is: -0.17; e.g. from expression (19), when actual GDP is greater than potential output, the unemployment gap falls by 0.17 percentage points. It implies that a decrease of unemployment gap of one percent is associated with an output gain of 5.88 (asymmetric Okun's law coefficient).

Although, models (16)-(18) are statistically significant, the estimated coefficients of models (19) and (20) captures nonlinear interactions between unemployment and output. Nevertheless, the results are quite similar.

4. Conclusions

The main purpose of this paper was to measure the impacts of unemployment on Mexico's output gap. Using quarterly data from 1993*Q*1 to 2016*Q*4, we estimated two time-varying coefficients models. Our principal conclusion is: Okun's law is a strong and stable relationship in Mexico. The econometric analysis suggests that when actual GDP is less than potential output, the unemployment gap rises by 0.17 percentage points. It implies that an increment of unemployment gap of one percent is associated with an output loss of -5.88 of potential output.

The upcoming question is: should Mexican Government wait for self-correction of the economy? Ros (2013) suggests that Government has policies it can use to reduce output gaps: fiscal policy (expenditures and taxation) and monetary policy (money supply and interest rates). Both fiscal and monetary policy changes aggregate demand without waiting for the economy to adjust itself.

However, be aware: Okun's law is a useful policy tool, but only if potential output and equilibrium unemployment are well-defined and properly measured.

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APPENDIX

A.1. Unit root tests

Table 1A summarizes the following unit root tests: Augmented Dickey-Fuller (A-DF), Dickey-Fuller GLS (DF-GLS), and Phillips-Perron (PP); t-statistic and p-values in parenthesis are reported. We reject the null hypothesis of unit root at the 95% level for all the variables.

Table 1A. Unit root test

	g^y	g^u	Δg^y	Δg^u
A-DF	-4.56 (0.00)	-2.43 (0.14)	-6.16 (0.00)	-6.10 (0.00)
DF-GLS	-4.50 (0.00)	-2.39 (0.02)	-6.19 (0.00)	-3.10 (0.00)
PP	-3.17 (0.03)	-3.09 (0.03)	-5.70 (0.00)	-7.03 (0.00)

A.2. Baseline and ARDL models: Goodness-of-fit and diagnostic statistics

Tables 2A-5A summarize the following Goodness-of-fit and diagnostic statistics: R-squared (R^2), Adjusted R-squared (R_A^2), Standard Error of the Regression (*s*), Jarque-Bera statistic (*JB*), Serial Correlation Lagrange Multiplier test (*LM*), Breusch-Pagan-Godfrey test (*BPG*), and White’s Heteroskedasticity test (*WH*); p-values in parenthesis are reported.

Table 2A. Model (15): Goodness-of-fit and diagnostic statistics, 1993*Q*1-2016*Q*4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
μ	0.10	0.03	2.85	0.01
g_t^y	-0.22	0.02	-14.05	0.00

$R^2 = 0.68$, $R_A^2 = 0.68$, *s* = 0.34, *JB* = 20.00 (0.00), *LM(1)* =110.64 (0.00), *LM(1-2)* = 54.78 (0.00), *LM(1-36)* = 3.22 (0.00), *BPG* = 29.20 (0.00), *WH* = 39.14 (0.00)

Table 3A. Model (16): Goodness-of-fit and diagnostic statistics, 1993*Q*2-2016*Q*4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Δg_t^y	-0.17	0.02	-8.86	0.00
d_{95Q3}	0.70	0.21	3.28	0.00
d_{00Q2}	0.69	0.21	3.20	0.00

$R^2 = 0.50$, $R_A^2 = 0.49$, *s* = 0.21, *JB* = 2.62 (0.27), *LM(1)* = 1.41 (0.24), *LM(1-2)* = 0.80 (0.45), *LM(1-36)* = 0.51 (0.98), *BPG* = 0.45 (0.72), *WH* = 1.10 (0.36)

Table 4A. Model (17): Goodness-of-fit and diagnostic statistics, 1993Q3-2016Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
g^u_{t-1}	0.84	0.06	14.19	0.00
g^y_t	-0.12	0.02	-5.83	0.00
g^y_{t-1}	0.01	0.03	0.37	0.71
g^y_{t-2}	0.09	0.02	4.01	0.00
d_{00Q2}	0.72	0.20	3.58	0.00

$R^2 = 0.90$, $R^2_A = 0.90$, s = 0.20, *JB* = 2.17 (0.34), *LM(1)* = 0.18 (0.67), *LM(1-2)* = 1.34 (0.27), *LM(1-36)* = 0.81 (0.74), *BPG* = 1.16 (0.33), *WH* = 1.76 (0.7)

Table 5A. Model (18): Goodness-of-fit and diagnostic statistics, 1993Q3-2016Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Δg^u_{t-1}	-0.06	0.09	-0.70	0.48
Δg^y_t	-0.13	0.02	-6.49	0.00
Δg^y_{t-1}	-0.10	0.02	-4.15	0.00
d_{00Q2}	0.77	0.21	3.77	0.00

$R^2 = 0.55$, $R^2_A = 0.54$, s = 0.20, *JB* = 1.82 (0.40), *LM(1)* = 0.27 (0.60), *LM(1-2)* = 0.30 (0.74), *LM(1-36)* = 0.73 (0.84), *BPG* = 0.36 (0.83), *WH* = 0.59 (0.76)

A.3. TVC models: Goodness-of-fit and diagnostic statistics

Tables 6A and 7A summarize the following Goodness-of-fit and diagnostic statistics: R-squared (R^2), Adjusted R-squared (R^2_A), Standard Error of the Regression (s), Bowman-Shenton statistic (*BS*), Box-Ljung statistic (*BJ*), and Heteroskedasticity test (*H*); p-values in parenthesis are reported. State vector analysis at period 2016(4).

Table 6A. Model (19): Goodness-of-fit and diagnostic statistics, 1993Q1-2016Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
μ_t	-0.33	0.29	-1.16	0.25
g^y_t	-0.17	0.02	4.15	0.00
d_{95Q3}	0.62	0.15	-3.32	0.00
d_{97Q1}	-0.65	0.19	3.53	0.00
d_{00Q2}	-0.69	0.20	-10.52	0.00

$R^2 = 0.90$, $R^2_A = 0.90$, s = 0.19, *BS* = 5.36 (0.06), *BJ(1-2)* = 0.68 (0.40), *BJ(1-12)* = 12.10 (0.36), *H(30)* = 0.70 (0.83)

Table 7A. Model (20): Goodness-of-fit, 1993Q1-2016Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ω_t	-0.01	0.02	-0.56	0.58
Δg_t^y	-0.17	0.02	-9.68	0.00
d_{95Q3}	0.72	0.20	3.61	0.00
d_{97Q1}	-0.61	0.20	-3.11	0.00
d_{00Q2}	0.70	0.20	3.52	0.00

$R^2 = 0.57$, $R_A^2 = 0.57$, s = 0.20, *BS* = 2.53 (0.28), *BJ(1-2)* = 1.11 (0.29), *BJ(1-12)* = 13.90 (0.24), *H(30)* = 0.72 (0.82)

A.4. Models comparison: Actual and fitted series

Figures 1A-2A plots actual and fitted series.

Figure 1A. Actual g^u and fitted series, 1994Q1-2016Q4

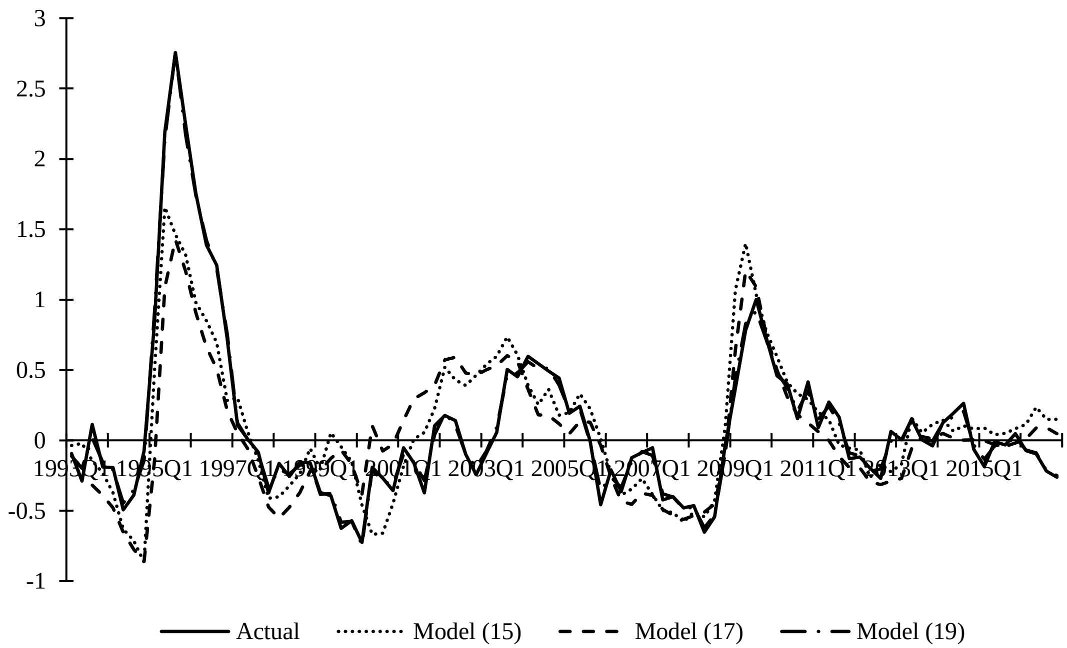

Figure 2A. Actual Δg^u and fitted series, 1994Q1-2016Q4